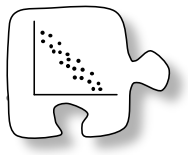
6.1.3 What is the relationship?



Identifying Correlation

When is it reasonable to make a prediction? For example, when you know the height of a tree, can you predict the size of its leaves? Or if you know the outdoor temperature for the day, can you predict the number of glasses of water you will drink during the day?

In Lesson 6.1.2 you found that some data sets were related and others were not. In this lesson you will look at different situations and decide if they show a relationship, and if that relationship allows you to make a prediction. As you work with your team today, use these focus questions to help direct your discussion:

When one value goes up, what happens to the other one?

Is there a relationship between one thing changing and the other changing?

Can I make a prediction?

6-21. Students in a science class have been experimenting with different factors that they think may affect how tall a plant will grow. Each team planted seeds in several pots using different experimental conditions.

With your team, read the questions that the students investigated. Write a team prediction (hypothesis) for the results for each experiment. Assume that all other variables are controlled (meaning that they will not affect the experimental outcome).

a. Does the amount of fertilizer affect the plant height?

b. Does how deep the seed is planted in each pot affect the plant height?

c. Does the number of seeds in each pot affect plant height?

d. Does the size of the pot affect plant height?

e. Does the hours of sunlight per day affect the plant height?

6-22. After three weeks, the teams measured the heights of their plants and recorded the data. The Team 1 data chart and the question they investigated are included below. On graph paper, make a scatterplot for the data gathered. Be sure that you:

- Clearly label your axes
- Mark the scale at equal intervals
- Title your graph appropriately (such as with the experimental question)

Team 1: Does the amount of fertilizer affect the plant height?

Amount of fertilizer over 3 week period (ml)	5	10	15	20	25
Height of plant (cm)	9	12	18	24	30

6-23. Your teacher will assign your team one of the remaining sets of data and will provide you with poster material. Prepare a scatterplot poster for your assigned set of data. Be sure your graph has a title and that the axes are correctly labeled. Also make sure that the points on your graph will be easily seen from across the room.

Team 2: Does the depth of seed in each pot affect the plant height?

Depth of seed in pot (cm)	3	6	9	12	15
Height of plant (cm)	24	21	18	12	6

Team 3: Does the number of seeds in each pot affect the plant height?

Number of seeds planted in pot	1	2	3	4	5
Height of plant (cm)	21	24	27	21	24

Team 4: Does the size of the pot affect plant height?

Diameter of pot (cm)	6	9	12	15	18
Height of plant (cm)	21	20	19	21	18

Team 5: Do the hours of sunlight per day affect the plant height?

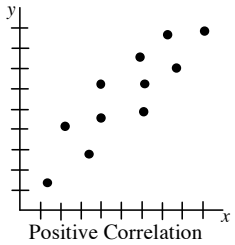
Amount of light per day (hours)	1	3	5	7	9
Height of plant (cm)	3	9	12	24	27

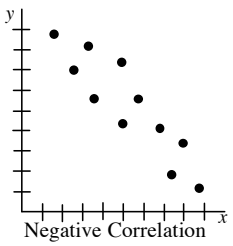
6-24. Examine your scatterplot for Team 1 (from problem 6-22) and the scatterplot posters created by your class. Then answer the following questions.

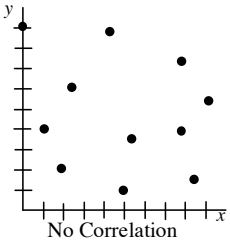
a. For each of the graphs of data, does there appear to be a relationship? If so, describe it by completing the sentence:

As _____ gets larger, then _____ gets _____.

b. In a scatterplot, if there appears to be no relationship between the variables, then the points in the scatterplot have **no correlation**. But if one variable generally increases as the other variable increases, there is said to be a **positive correlation**. If one variable generally decreases as the other variable increases, there is said to be a **negative correlation**. See some examples below.







Positive Correlation Negative Correlation No Correlation

Review each of the graphs of the plant experiment data and decide if there is a positive correlation, a negative correlation, or no correlation.

c. An **outlier** is a piece of data that does not seem to fit into the pattern. Do there appear to be any outliers in any of the scatterplots?

d. Now go back and look at your team predictions for each question (problem 6-21). Were your predictions accurate? Explain your **reasoning**.

6-25. When there is a correlation, predictions can be made. One way to help make predictions is to draw a trend line or curve for the data.

a. Find your graph of Team 1's data (from problem 6-22). Work with your team to draw a straight line that represents the trend of the data on this graph. Remember that the line does not need to intersect each of the points.

b. Now use your trend line to predict the height of the plant when 12 milliliters of fertilizer is given to the plant over a 3-week period.

c. How much fertilizer would you predict is needed (over the 3-week period) to have the plant grow 26 cm?

6-26. LEARNING LOG

In your Learning Log, describe how a graph can help you tell if there is a relationship between two sets of data. That is, what does it look like if there is a correlation? What does it look like if there is no correlation? Title this entry, "Correlations," and label it with today's date.

Review & Preview

6-27. HOW MUCH IS THAT CAR?

Nate and Nick were talking about cars. Nate claimed that cars with more horsepower were more expensive than cars with less horsepower. His reason was that his 300 horsepower sports car was worth more than Nick's 120 horsepower small economy car. To investigate Nate's claim, the boys looked up information about their friends' cars. The table at right shows their results.

Nick and Nate's Data

Horsepower	Price (Thousands)
500	$38
160	$16
453	$23
505	$23
228	$30
311	$26
335	$15
197	$40

a. Does the information in the table support Nate's claim that cars with more horsepower cost more? Is there a relationship between horsepower and the price of a car?

b. Set up a graph and plot the points from the table. Now do you believe Nick's claim? Explain your **reasoning**.

6-28. Evaluate the inequality $x + 4 < 2x - 3$ for the following values of x. Decide if the given value of x makes the statement true or false. Show all of your work.

a. $x = 4$ b. $x = -2$ c. $x = 7$ d. $x = 9$

6-29. Evaluate the following expressions.

a. $7x + 8$ when $x = 9$ b. $6(y - 11)$ when $y = -6$

c. $45 - 5m + 7$ when $m = -4$ d. $-2t + 9$ when $t = -20$

6-30. Simplify each expression.

a. $-\frac{4}{5}+\frac{7}{12}$ b. $\frac{5}{9}+(-\frac{1}{4})$ c. $-\frac{3}{7}\cdot\frac{11}{12}$ d. $-1\frac{2}{3}\cdot\frac{4}{5}$

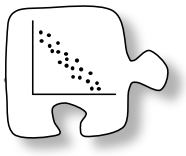
6-31. Here are the heights (in inches) of hollyhocks (tall, slender flowering plants) that are growing in the park: 10, 39, 43, 45, 46, 47, 48, 48, 49, 50, and 52.

a. Find the median.

b. Find the quartiles.

c. Make a box-and-whisker plot of the data.

6.1.4 What is the relationship?



Introduction to Linear Rules

In Lessons 6.1.2 and 6.1.3, you made scatterplots and examined at how data was related. With your team, you will now explore how some kinds of correlations in scatterplots can be used to help make predictions. So far, the data you have graphed has been a collection of points that you have gathered from the world around you. Today we will begin to collect data from a mathematical situation and attempt to use this data to make predictions. This data, when displayed on a graph, will be like a scatterplot in some ways and different in other ways.

6-32. COLLECTING DATA

You and a partner will be given a short time to generate a list of number pairs that add to ten. For example, 3 plus 7 equals 10, so the number pair would be $(3, 7)$. Use a table like the one below to record your number pairs.

Number 1													
Number 2													

As you generate your list, think about kinds of numbers you can use.

- Do the numbers have to be positive?
- Do they have to be integers?
- What is the largest number you can use? What is the smallest?
- How many number pairs are possible?

6-33. ANALYZING THE DATA

Follow the directions from your teacher to create a graph of the data. Then, with your team, discuss the questions below and record your answers.

a. What do all of the number pairs have in common?

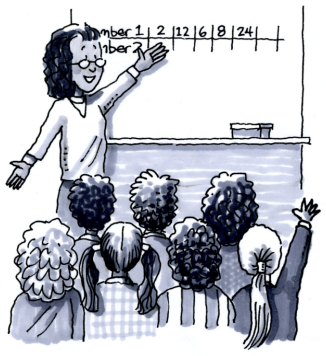
b. Explain the **strategy** you used to find your number pairs.

c. How many points could there be on the graph?

d. How is this relationship different than the scatterplots that you have made in Lessons 6.1.2 and 6.1.3? How is this relationship the same?

6-34. Now you will play what are known as Silent Board Games. Your teacher will put an incomplete $x \rightarrow y$ table on the overhead or board. For each:

- Copy the table, study the input and output values, and look for a relationship.
- Once you know the relationship between x and y, use the relationship to add the missing values to your chart.
- Finally, write the rule in words and symbols that finds each y-value from its x-value.



6-35. In Chapter 1, your team looked for characteristics that sets of objects had in common. The points below were one of those sets of objects. Use what you learned about finding relationships during the Silent Board Game to describe the relationship between x and y in the points below. Then write a rule of the form $y =$ _____ .

(3,5) (98,100) (−6,−4) (11,13)

6-36. Look for a relationship between the numbers in each table below. Complete the last two entries for each table. Describe any relationships you notice.

a.

Number 1	2	12	6	8	24	–4	1	
Number 2	3	18	9	12	36	–6		30

b.

Number 1	2	–5	7	4	0	–1	3	
Number 2	5	–9	15	9	1	–1		11

c.

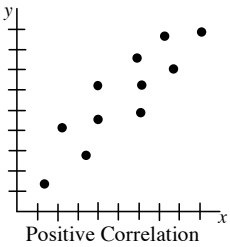
Number 1	–3	5	–5	0	2	–2		6
Number 2	1	–7	3	–2	–4	0	2	

MATH NOTES

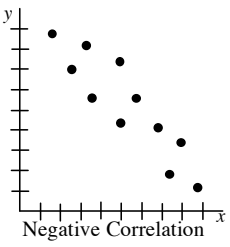
METHODS AND MEANINGS

Correlation and Trend Lines

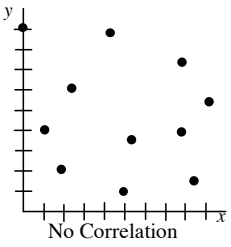
In a scatterplot, if one variable increases as the other variable increases, there is said to be a **positive correlation**. If one variable increases as the other variable decreases, there is said to be a **negative correlation**. If there is no relationship between the variables, then the points in the scatterplot have **no correlation**. An example of each situation is illustrated below.



Positive Correlation

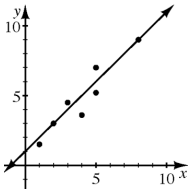


Negative Correlation



No Correlation

A **trend line** is a line that represents the general tendency of a set of data. It does not need to intersect each data point. Rather, it needs to approximate the data. A trend line looks and "behaves" like the data, as shown in the example at right.



Review & Preview

6-37. Consider the following table.

x	–3	0.5	0	2	4
y	–1	2.5	2	4	6

a. What is the rule for the table?

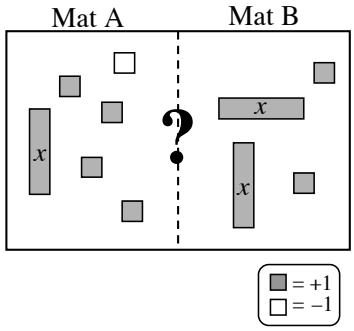
b. Explain your **strategy** for finding the rule for this table.

6-38. Write an expression for each mat at right.

a. Simplify each mat to determine which expression is greater, if possible.

b. If $x = 3$, would your answer to part (a) change? Explain.

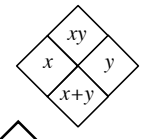
c. If $x = -2$, would your answer change? Explain.

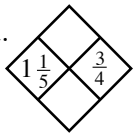


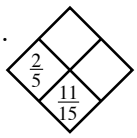
6-39. **Multiple Choice:** Which of the following expressions could be used to find the average (mean) of the numbers k, m, and n?

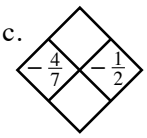
A. $k+m+n$ B. $3(k+m+n)$ C. $\frac{k+m+n}{3}$ D. $3k+m+n$

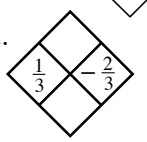
6-40. Copy and complete each of the Diamond Problems below. The pattern used in the Diamond Problems is shown at right.



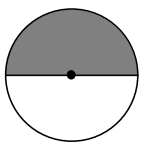
a. 

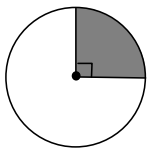
b. 

c. 

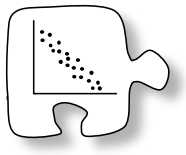
d. 

6-41. Find the area of the shaded portion of each circle.

a. 

b. 

6.1.5 What is the pattern?



Intercepts

In past lessons you looked at data from several situations. Sometimes the data appeared to be random and sometimes it had a predictable pattern. When data has a rule-based pattern, we can make predictions by finding other data points that fit the pattern.

6-42. SILENT BOARD GAME

Today you will continue to play the Silent Board Game. Your teacher will put an incomplete $x \rightarrow y$ table on the overhead or board. Study the input and output values and look for a pattern. Then write the rule in words and symbols that finds each y-value from its x-value.

6-43. CLEANING THE TANK, Part One

Roland and Alli work at the Seattle Aquarium. Part of their job is to empty the tank that holds a Giant Pacific Octopus in order to clean the glass and rocks. It is quite a job!

On one cleaning day, Roland carefully transferred the octopus to another tank. He then opened the drain to remove the water. As the water drained, the water level (its height in the tank) lowered at a rate of 2 feet per hour.

When Alli arrived at work that afternoon, she noted that there was still 10 feet of water in the tank.

a. Alli started a table to record the height, as shown below. Copy her table on your paper and complete the missing values. Then graph the data on graph paper.

x (hours after Alli arrived)	–5	–1	0	1	2	3	5	x
y (height in feet)		12	10					$y =$

b. What do the x-values in Alli's table represent? For example, what does $x = 3$ represent? What does $x = -1$ represent?

c. How does the graph tell you what is happening between 2 and 3 hours after Alli got to work? Use the graph to find the water level 2.5 hours after she got to work.

Problem continues on next page. →

6-43. *Problem continued from previous page.*

d. Does it make sense to connect the points? Why or why not?

e. What will the height of the water be two hours after Alli gets to work? What was the height of the water two hours before she got to work?

f. If the water level is 20 feet when the tank is full, how many hours did it take to drain it? How many hours before Alli got to work did Roland open the drain? Explain how you found your answer.

6-44. INTERPRETING INTERCEPTS

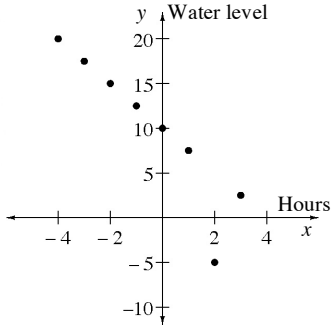
The point where a line crosses the x- or y-axis on a graph often gives useful information about a situation. Examine your graph from problem 6-43.

a. The point where the line crosses the x-axis is called the ***x*-intercept**. What are the coordinates of this point on your graph? What does that point tell you about the water in the tank?

b. The point where the line crosses the y-axis is called the ***y*-intercept**. What are the coordinates of this point? What information does it give about the water in the tank?

6-45. After Alli graphed the data (shown at right), she became concerned because her graph no longer made a straight line.

Giant Pacific Octopus Tank



a. At what x-value did Alli make her mistake? What y-value should she have?

b. Explain to Alli how she could have recognized her mistake.

6-46. **Additional Challenge:** Alli's data table from problem 6-43 was based on when she arrived at work. However, if Roland had made the data table instead, starting with when he opened the drain, his table would have been slightly different. Assume that the tank originally was filled to a height of 20 feet.

a. On your paper, create a new table that records the height of the water for the number of hours after the drain was opened. Start with $x = 0$ hours and include each hour until the tank was empty.

b. If you graphed Roland's data from part (a), how would this graph be the same or different than Alli's graph from problem 6-43? **Visualize**.

c. Graph Roland's data from part (a). Compare Roland's graph with Alli's graph. Write down your observations.

6-47. LEARNING LOG

In your Learning Log, explain how you can make predictions with scatterplots and linear graphs. Is one kind of graph easier to use to make predictions? Can you always make a prediction with a graph? Label this entry with today's date and title it "Scatterplot and Linear Graph Predictions."

METHODS AND MEANINGS

Making a Complete Graph

MATH NOTES

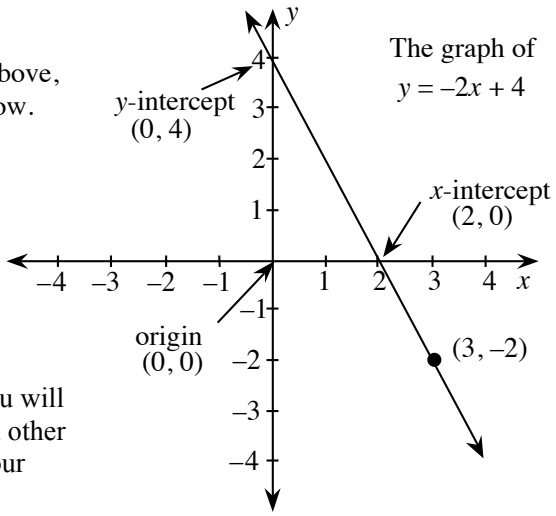
A complete graph has the following components:

- *x*-axis and *y*-axis labeled, clearly showing the scale.
- Equation of the graph near the line or curve as a label.
- Line or curve extended as far as possible on the graph.
- Coordinates of special points stated in (x, y) format.

x	–1	0	1	2	3
y	6	4	2	0	–2

Tables can be formatted horizontally, like the one above, or vertically, as shown below.

x	y
–1	6
0	4
1	2
2	0
3	–2
4	–4

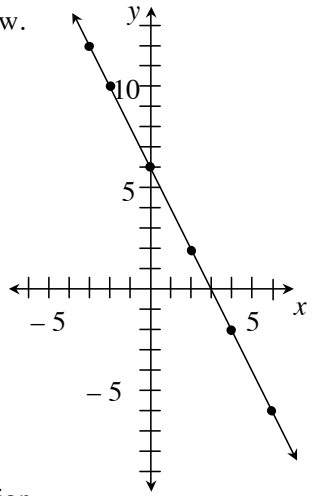


The graph of $y = -2x + 4$

Throughout this course, you will continue to graph lines and other curves. Be sure to label your graphs completely.

Review & Preview

6-48. Use the graph at right to add points to the table below.



x	
y	

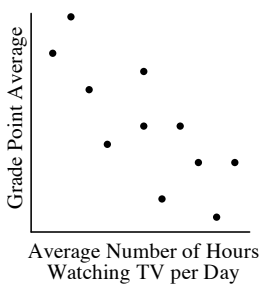
a. Write the rule in words.

b. Explain how to use the table to predict the value of y when x is -8.

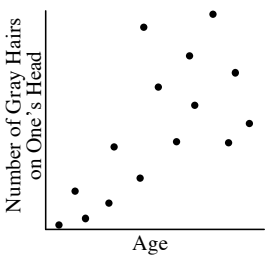
6-49. For each scatterplot below:

- Determine if there is a "trend" or correlation in the points. If there is, describe it.
- Label each graph as showing a positive correlation, negative correlation, or no correlation.

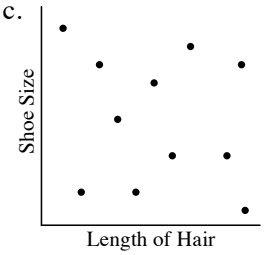
a.



b.



c.



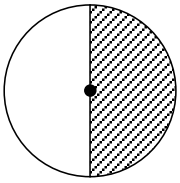
6-50. Evaluate the expressions $3x - 2$ and $4x + 4$ for the following values of x. When you have found the value for each expression, write a statement using $<$, $>$, or $=$ that shows how the two values are related.

a. $x = 0$ b. $x = -6$ c. $x = 5$ d. $x = -2$

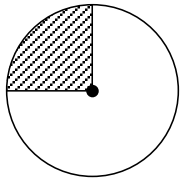
6-51. Draw a trapezoid. Label the top base 6 cm and the height 4 cm. If the area of this trapezoid is 36 square cm, what is the length of the other base? You may find it helpful to use the 5-D Process.

6-52. Find the area of each shaded portion, called a **sector**, in the circles below.

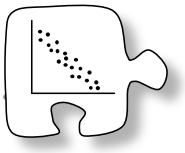
a. The diameter is 3 feet.



b. The radius is 12 ft.



6.1.6 When does a graph make a straight line?



Tables, Linear Graphs, and Rules

In the previous lesson you made a graph showing how the water level in a tank changed as it was being drained at a constant rate. You noticed that the graph made a straight line. Today you will complete a table and graph what happens as the Giant Pacific Octopus tank is being filled. As you work with your team, think about the following questions:

How is the line changing?

What information can we get from a rule?

When does a graph form a straight line? When does it not?

6-53. SILENT BOARD GAME

Today you will continue to play the Silent Board Game. Your teacher will put an incomplete $x \rightarrow y$ table on the overhead or board. Study the input and output values and look for a pattern. Then write the rule in words and symbols that finds each y-value from its x-value.